REBOUND OF A RIGID BODY FALLING ON AN ELASTIC HALF-SPACE

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 7, No. 5, pp. 173–174, 1966

An approximate solution of the problem of the rebound of a rigid massive body with a smooth surface falling on an elastic half-space is derived, and the results of calculations are presented.

§1. A rigid body of mass m with a smooth convex surface falls (without rotation) on an elastic half-space with a velocity v_0 directed perpendicularly to the half-space boundary. The assumed form of the

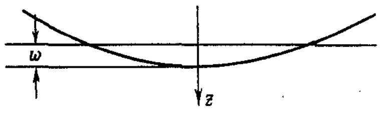

Fig. 1

body is such that the contact zone has a fixed center of symmetry, while the center of mass of the body is on a normal to the half-space drawn through the center of the contact zone. There is no friction between the body and the medium.

Let us denote by w(t) the displacement of the body mass center relative to the position it occupied at the moment of contact with the medium. It will be assumed that w is small in comparison with the principal radii of the body surface curvature in the contact zone.

The problem consists in determining w(t) and the rebound velocity if rebound takes place. For w(t) we have the following equation:

$$mw^{\cdot\cdot} = F \qquad (w(0) = 0,\ w^{\cdot}(0) = v_0). \tag{1.1}$$

Here F is the resultant of the normal stresses in the contact zone which depends on w. To determine F it is necessary to solve a dynamic problem of a punch with a smooth surface being forced into an elastic half-space in accordance with an arbitrary law w(t). The solution of this problem is not known. Let us make an approximate assumption that

$$F = F_1 + F_2 \qquad (F_1 = -\rho a \Omega(w) w^{\cdot},\ F_2 = -kw^{\alpha}). \tag{1.2}$$

Here ρ is the density of the medium; a is the longitudinal wave velocity; Ω is the contact zone area; F_2 is the resultant of normal stresses in the corresponding static problem [1]; and k and α are constants depending on the shape of the body and parameters of the medium. It can be shown that in the initial indentation stage, when the velocity of the contact zone edges is larger than a, $F = F_1$. In this case the formula (1.2) will be a good approximation of the accurate expression for the

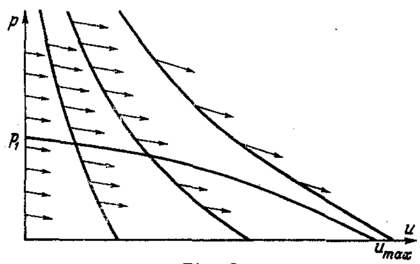

Fig. 2

resultant force in the initial stages of the process, when the rate of indentation is rapid and the displacement small, and also in the final stages (before the movement ceases) when the indentation rate is slow. An analagous approximation is obtained from the accurate solution of the problem of a plate being forced into an elastic half-space [2]. From Eqs. (1.1) and (1.2) we obtain

$$mw^{\cdot\cdot} + a\rho\Omega(w) w^{\cdot} + kw^{\alpha} = 0. \tag{1.3}$$

The condition of discontinuation of contact between the body and the medium is that F = 0, or that

$$w^{\cdot\cdot} = 0. \tag{1.4}$$

If the falling body is a sphere of a radius R, we have [1]

$$\Omega(w) = 2\pi R w, \qquad k = 4E\sqrt{R} / 3(1 - \nu^2). \tag{1.5}$$

Let us introduce dimensionless values $u = w/R$, $\tau = at/R$. From Eq. (1.3) we obtain for u the following nonlinear equation

$$\begin{gathered} u'' + 0.5\varkappa_1 u (u' + \varkappa_2 \sqrt{u}) = 0, \\ \varkappa_1 = 3M, \quad M = \rho / \rho, \\ \varkappa_2 = 8\gamma^2 (1 - \gamma^2) / 3\pi), \\ u(0) = 0, \qquad u'(0) = u_0. \end{gathered} \tag{1.6}$$

Here ρ_0 is the average density of the sphere; γ is the ratio of velocities of the transverse and longitudinal waves; and prime indicates differentiating in respect to τ.

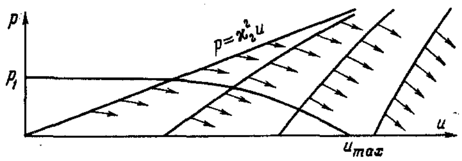

Fig. 3

§2. Equation (1.6) allows for its order to be lowered. Denoting $p = (u')^2$, we obtain

$$\frac{dp}{du} = -\varkappa_1 u (\varkappa_2 \sqrt{u} \pm \sqrt{p}), \qquad p(0) = u_0^2. \tag{2.1}$$

Here the plus sign is used when the body is moving into the medium and the minus sign for the movement in the opposite direction. The rebound condition assumes the form

$$dp/du = 0. \tag{2.2}$$

Figure 2 shows the direction field and the integral curve of Eq. (2.1) for the inward movement of the body, Fig. 3 showing the corresponding data for the outward movement. The integral curves of the outward motion do not continue beyond a straight line $p = \varkappa_2^2 u$, since points on this line correspond to states when the contact between the body and the medium discontinues (rebound).

It is interesting to note that the contact is discontinued $u \neq 0$, i.e., before the points of the medium revert to their undisturbed state. Experiment showed that, other conditions being equal, the larger the sphere radius the slower is the rebound velocity or, which is the same, that a massive body rebounds with a higher velocity which increases with increasing $\gamma = \sqrt{\mu / (\lambda + 2\mu)}$.

In Fig. 4 the coefficient of restitution v_1/v_0 (where v_1 is the rebound velocity) is plotted against the relative fall velocity v_0/a.

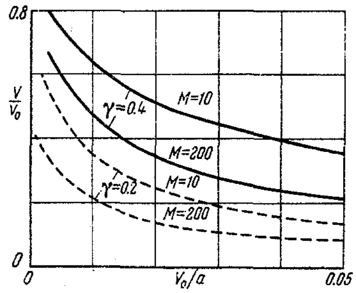

Fig. 4

It will be seen that at small v_0/a the coefficient of restitution is near to one, i.e., only a small proportion of the energy of the body is transferred to the medium. At $v_0/a = 0.05$, however, the proportion

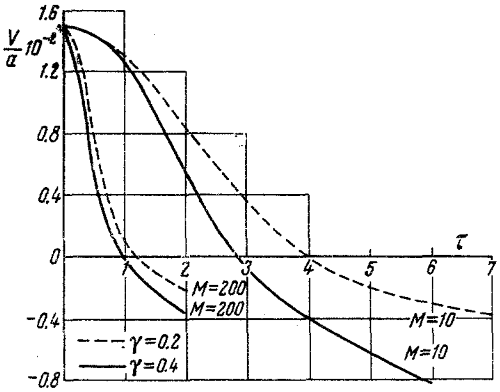
1.6
V/a 10^-2
1.2
0.8
0.4
0
-0.4
-0.8
τ
1
2
3
4
5
6
7
M=200
M=200
M=10
M=10
γ=0.2
γ=0.4

Fig. 5

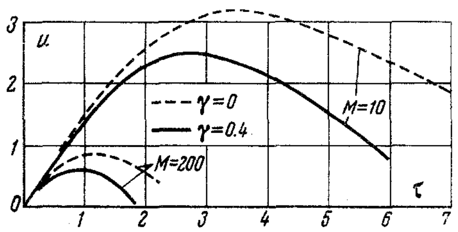
3
u
2
1
0
γ=0
γ=0.4
M=10
M=200
τ
1
2
3
4
5
6
7

Fig. 6

of energy represented by the vibrations of the half-space is 85–99% (for the magnitude of M and γ used in the calculations).

In the range of v_0/a between 0.001 and 0.05 (for the values of M and γ used) the energy of the vibrations of the medium and the energy accumulated in the sphere at the moment of rebound are of a similar order of magnitude.

This is in agreement with results reported in [3, 4] where it is asserted that the impact may be regarded as quasi-static only at

$$(v_0 / a)^{1/5} \ll 1.$$

Graphs plotted in Fig. 5 represent the dependence of v/a on the relative time $\tau = at/R$, while in Fig. 6 the relative displacement u/R is plotted against τ (in both cases for $v_0/a = 0.015$ and $\gamma = 0.2$ and 0.4).

Comparison of calculations with experiments in which a solid body was dropped on dry ground shows that calculated data will be the upper limit of such impact parameters as the rebound velocity and overload, i.e., the theory expounded above predicts a more "rigid" impact; the difference is mainly due to plastic strains produced by impact in the ground.

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10 May 1966 Moscow